COMPUTATION OF DEPTH OF PRECIPITABLE WATER IN A COLUMN OF AIR

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[Weather Bureau, Washington, D. C., December 1938]

It has been found that one of the most significant quantities in hydrometeorological studies is Wp, the depth of precipitable water in a column of air. After some experimentation, the following simple tabular method of computing the value of Wp from aerological observations was developed for use in the Hydrometeorological Research Section.

Let M_w=total mass of water vapor,
h=height,
ρ_w=density of water vapor,
ρ=density of air,
g=acceleration of gravity,
q=specific humidity,
P=pressure in mb,
Wp=depth of precipitable water (inches).

For a column of water vapor with the horizontal cross-section considered as unity:

$$dM_w=\rho_w dh \tag{1}$$

$$M_w=\int_0^h \rho_w dh \tag{2}$$

$$dp=-\rho g dh \tag{3}$$

combining (2) and (3)

$$M_w=-\int_{P_0}^{P_h}\frac{\rho_w dp}{\rho g} \tag{4}$$

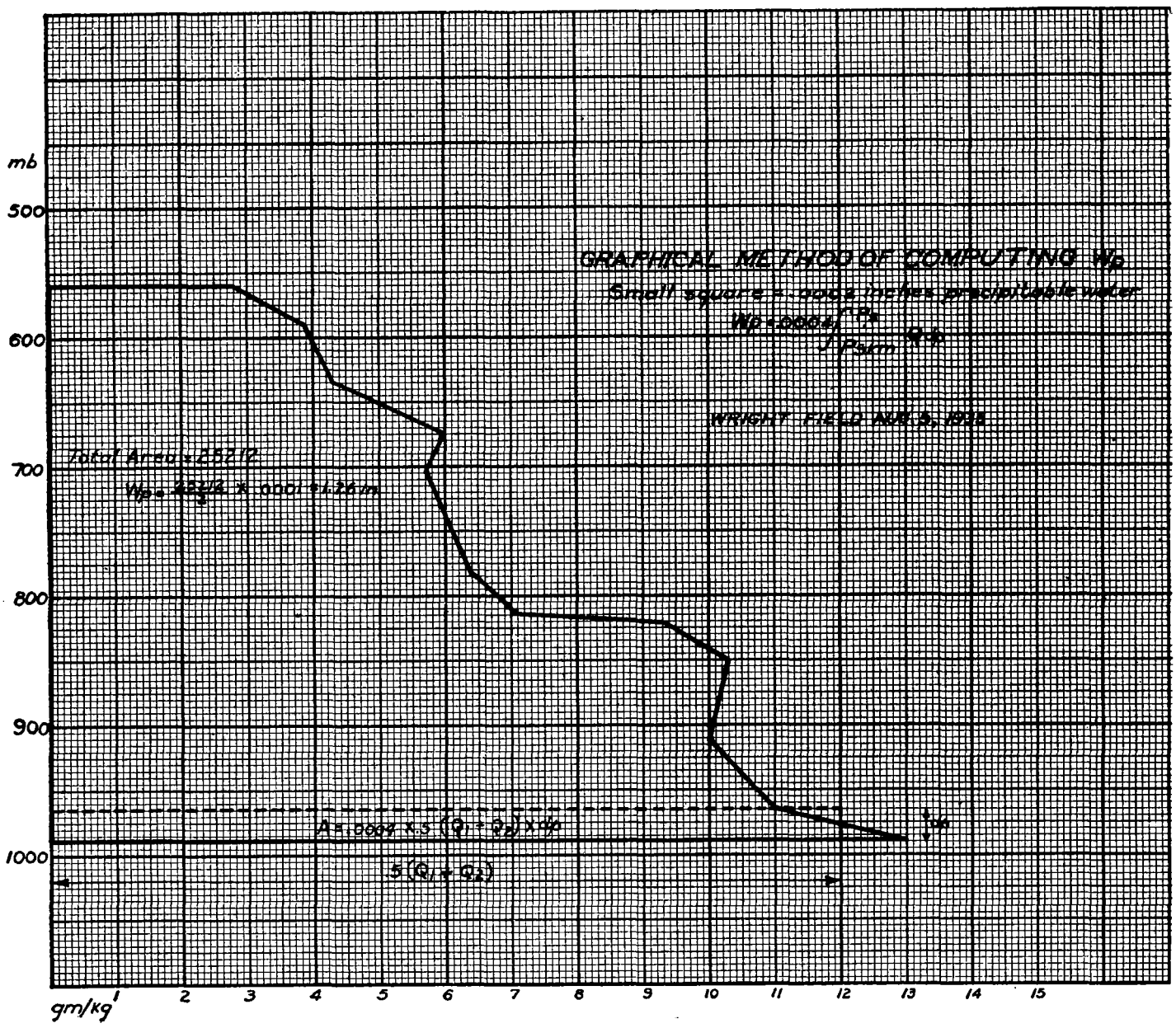

FIGURE 1.—Graphical method of computing precipitable water.

g can be considered a constant with negligible error.

$$M_w = -\frac{1}{g}\int_{P_0}^{P_h}\frac{\rho_w dp}{\rho} \tag{5}$$

since $q = \frac{\rho_w}{\rho}$

$$M_w = -\frac{1}{g}\int_{P_0}^{P_h} q\,dp \tag{6}$$

Reversing "direction" of integrations

$$M_w = \frac{1}{g}\int_{P_0}^{P_0} q\,dp \tag{7}$$

Figure 2.

COMPUTATION OF DEPTH OF PRECIPITABLE WATER

(Ohio)

Station Wright Field Elevation ____ Computed by SBS Date December 2, 1938.

Date of Observation August 5, 1935. Checked by JRR Date December 2, 1938.

Type of Instrument Aerometeorograph File No. 93508

Levels Sfc.	e	P	q	dp	Q_{2m}	A	dwp
1.		989	13.0				
				24	24.0	576	
2.		911	10.0				
				54	21.0	1134	
3.		851	10.3				
				60	20.3	1218	
4.		824	9.3				
				27	19.6	529	
5.		814	7.1				
				10	16.4	164	
6.		776	6.3				
				38	13.4	509	
7.		706	5.7				
				71	12.0	852	
8.		676	6.0				
				30	11.7	351	
9.		633	4.3				
				42	10.3	433	
10.		592	3.9				
				41	8.2	336	
11.		562	2.8				
				30	6.7	201	
12.							
13.							
14.							
15.							
16.							
17.							
18.							
19.							
20.							
					Total	6303	1.26

e = vapor pressure in mb. dp = P_1 minus P_2, etc.

P = pressure in mb. Q_{2m} = $q_1 + q_2$, etc.

q = specific humidity g/kg. A = Qdp

0.0002 A = W_p, depth in inches of precipitable water.

Dimensional formulas may be expressed as follows:

q=non-dimensional ration.

$$P = \frac{ML}{L^2T^2}.$$

$$g = \frac{L}{T^2}.$$

$$\frac{Pq}{g} = \frac{MLT^2}{L^2T^2} = \frac{M}{L^2}.$$

In the C. G. S. system this could be expressed as $\frac{gm}{cm^2}$. Since 1 cc. of water has a mass of 1 gram this becomes $\frac{cm^3}{cm^2}$ or cm. In order to evaluate the constant which would make the result expressible in inches of precipitable water, the following substitutions must be made:

g=980 cm/sec^2.

1 mb=1,000 $\frac{\text{dynes}}{cm^2}$.

1 inch=2.54 cm.

1 gm./kg. = $\frac{1}{1000}$ gms./gm.

$$\frac{1000}{980\times 2.54\times 1000} = .0004.$$

Figure 3

COMPUTATION OF DEPTH OF PRECIPITABLE WATER

(Tennessee)

Station Murfreesboro Elevation 174 M. Computed by SBS Date December 5, 1938.

Date of Observation May 26, 1935 Checked by JRR Date December 6, 1938.

Type of Instrument ____ File No. ____

	Levels Sfc.	e	P	q	dp	Q_{2m}	A	dwp
		9.0	1000	5.6				
500 M.	1.	7.9	964	5.1				
					36	10.7		.077
1000 M.	2.	6.4	907	4.4				
					57	9.5		.108
1500 M.	3.	5.2	850	3.8				
					57	8.2		.093
2000 M.	4.	4.0	799	3.1				
					51	6.9		.084
2500 M.	5.	2.4	752	2.0				
					47	5.1		.048
3000 M.	6.	1.9	706	1.7				
					46	3.7		.034
4000 M.	7.	0.9	621	0.9				
					85	2.6		.044
5000 M.	8.	0.6	547	0.7				
					74	1.6		.024
	9.							
	10.							
	11.							
	12.							
	13.							
	14.							
	15.							
	16.							
	17.							
	18.							
	19.							
	20.							
						Total		.51

e = vapor pressure in mb. dp = P_1 minus P_2, etc.

P = pressure in mb. Q_{2m} = $q_1 + q_2$, etc.

q = specific humidity g/kg. A = Qdp

0.0002 A = W_p, depth in inches of precipitable water.

Therefore $Wp = .0004\ q\ dp$ inches.

where q is expressed in gm./kg.

and dp in millibars.

Furthermore, it has been found that for practical purposes the precipitable water present from the surface to 5 kilometers above sea level may be considered as equal to the total amount of precipitable water, since the amount above that elevation is very small.

Thus $$Wp = .0004\int_{P_{5km}}^{P_s} q\,dp \tag{8}$$

It is possible to approximate the value of the above integral graphically by plotting significant values for q against P on linear cross section paper and computing the area under the curve.

To illustrate, the following data for Wright Field, Dayton, Ohio, August 5, 1935, were thus plotted (fig. 1).

P	q
989	13.0
965	11.0
911	10.0
851	10.3
824	9.3
814	7.1
776	6.3
705	5.7
675	6.0
633	4.3
592	3.9
562	2.8

P_n, P_{n-1}=pressure at any significant level and pressure at next lower significant level, respectively.

Q_n, Q_{n-1}=specific humidity at P_n and P_{n-1}, respectively.

From figure 1 it will readily be seen that the value of $A_{n,\,n-1}$, is equivalent to the area of a rectangle whose sides are

$$P_{n-1}-P_n \text{ and } .5(Q_{n-1}+Q_n)$$

$$A_{n,\,n-1}=.5(P_{n-1}-P_n)(Q_{n-1}+Q_n)$$

and $$Wp_{n,\,n-1}=.5\times.0004(P_{n-1}-P_n)(Q_{n-1}+Q_n)$$

$$=.0002(P_{n-1}-P_n)(Q_{n-1}+Q_n)$$

and $$Wp \quad =\sum .0002\ (P_{n-1}-P_n)(Q_{n-1}+Q_n)$$

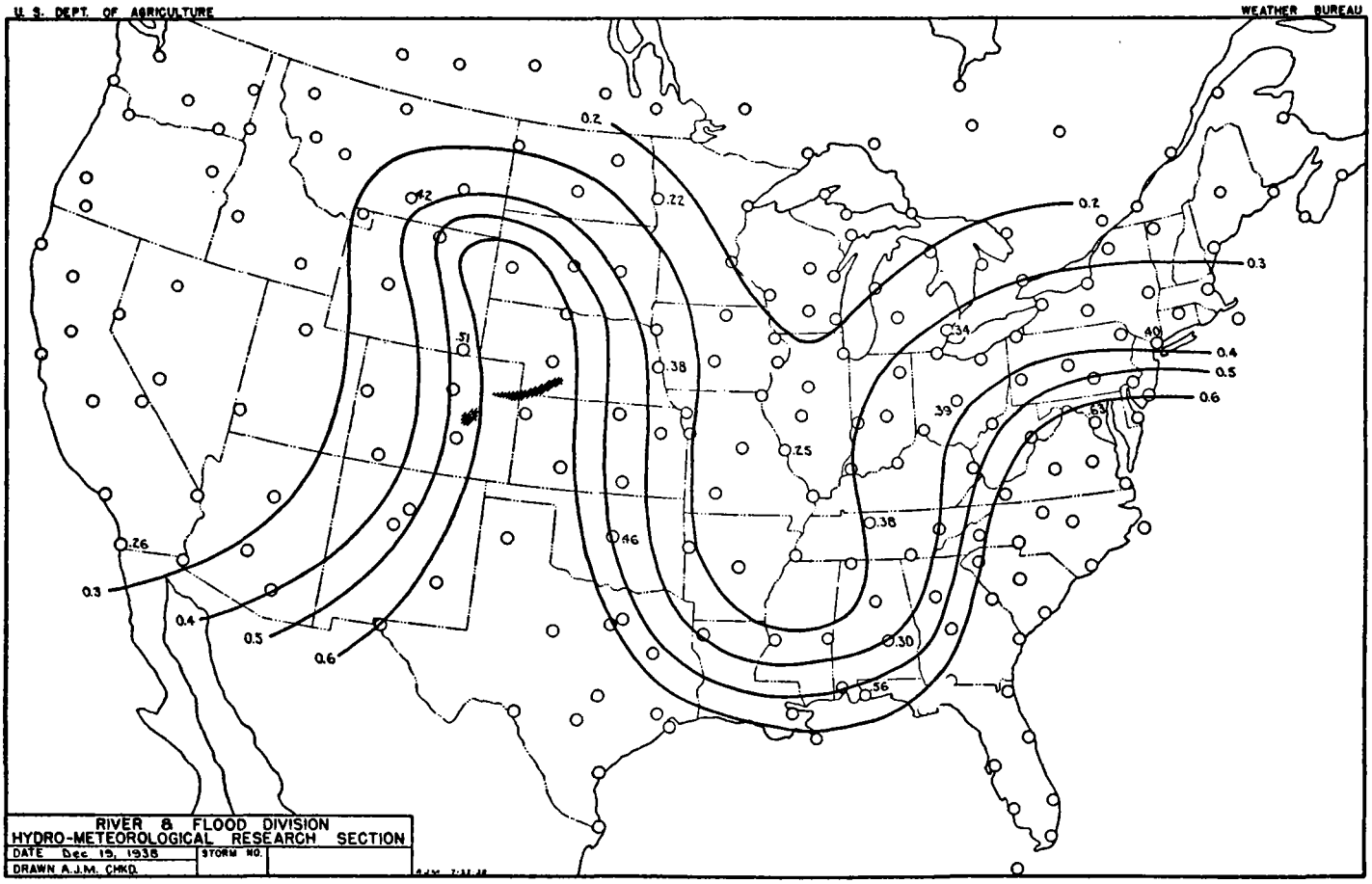

FIGURE 4.—Distribution of precipitable water from 2 to 5 kilometers on the morning of May 31, 1935, during the intense rain which caused the Republican River flood. Shaded portion shows the area of flood rains.

The scale in figure 1 was chosen for convenience. Since one small scalar element equals 0.1 gm./kg. and 5 millibars, it will be seen that one small square of area is equivalent to ½ equation units. Thus if A_t represents the total number of small squares, then

$$\frac{.5\times4}{10{,}000}A_t=\text{inches precipitable water,}$$

or

$$\frac{.0004}{2}A_t=\text{inches precipitable water.}$$

Let $A_{n,n-1}$=an area on the graph between any two significant pressures expressed in equation units. (p=1 *mb.*, q=1 gm./kg.).

If, instead of being plotted, the data were entered on a suitable form, the result is obtainable by means of very simple calculations. Figure 2 shows such a form.

In column P are entered pressures at significant levels. In column Q, specific humidities for corresponding levels. In column dp are the successive values for $P_{n-1}-P_n$. In column Q_{2m} are the successive values for $Q_{n-1}+Q_n$. Column A is the product of dp and Q_{2m}. The final result is the sum of column A multiplied by the constant .0002.

In some cases the values of Wp between various standard elevations are required. For this purpose it is more convenient to use standard level data from Weather Bureau Form 1109, instead of significant level data. The error involved in using standard level data is very small.

For this purpose column dWp has been provided. This column is simply the product of A and the constant .0002. In actual practice it is unnecessary to use both column A and dWp together, since dWp may be entered directly by multiplying dp by twice column Q_{2m} and setting decimal places properly. Three decimal places are sufficient. Column dWp may then be summed up and the third decimal place dropped. This will give Wp directly (fig. 3).

Column e is provided for cases where specific humidity is not given, but must be computed from vapor pressure, e.

It is not the purpose of this article to discuss the use of the quantity Wp. However, a chart showing the daily distribution of precipitable water from 2 to 5 kilometers over the United States is herein presented for general interest.

BIBLIOGRAPHY

[RICHMOND T. ZOCH, *in Charge of Library*]

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RECENT ADDITIONS

The following have been selected from among the titles of books recently received as representing those most likely to be useful to Weather Bureau officials in their meteorological work and studies:

Aguilar, Félix.

Reparación del aparato cuadripendular Askania no. 81952 del Instituto geográfico militar y determinación de los coeficientes de densidad y de temperatura de los péndulos de invar. La Plata. 1936. 17 p. tables, diagr. 29 cm. (Universidad nacional de La Plata. Publicaciones del Observatorio astronómico. Serie geodésica. Tomo I, no. I.)

Aircraft engineering; an international journal devoted to the science and practice of aeronautics. v. 7, no. 81. Nov., 1935. London. illus., diagrs. 35 cm. monthly.

Andreesen, Alfred.

Sturmfluten an der deutschen Nordseeküste. [Berlin?] 1937. 72 p. maps, tables, diagrs. 23 cm. (Leben und Arbeit. XXX. Jahrgang 1937. Mai 1937. 1. Heft.)

Ashbel, D.

[Miscellaneous papers. Jerusalem, Tel-Aviv.] 1936–1937. v. p. plate, tables, diagrs. 26 cm.

———

The climate of Palestine. Summaries for the year 5695 (1934–1935). Jerusalem. [1936?] 28 p. tables.

———

Evaporation in Palestine. Tel-Aviv. 1937. 21 p. tables, diagrs. (Reprinted from "The Palestine journal of botany and horticultural science." v. 1, no. 3, Rehovot. April, 1937.)

———

The fogs which ascend from the Dead Sea to the mountains of Jerusalem. [London. 1937.] 4 p. pl., diagrs. (Reprinted from The meteorological magazine. March, 1937.)

———

The intensity of the ultra-violet rays in Palestine. Jerusalem. 1936. p. 295–312. tables. (Folia ophthalmologica orientalia. v. 2, fasc. 4. 1936.)

———

On the importance of dew in Palestine. Jerusalem. 1936. p. 316–321. tab. (Reprinted from the Journal of the Palestine oriental society. v. 16. 1936.)

Barton, Samuel Goodwin, & Barton, William H., jr.

A guide to the constellations. London & New York. [°1935.] x, 74 p. incl. illus., charts, diagr. 31 cm. (Half-title: McGraw-Hill astronomical series. E. A. Fath, consulting editor.) 2d ed. Bibliography: p. 70.

Bellonte, Maurice.

La navigation aérienne commerciale sur l'Atlantique nord. [Paris. 1935.] p. 51–59. maps, tables. 30 cm. [Photostated from L'Aérophile. Mars, 1936.]

Bergeiro, José M.

El vegetal en sus relaciones con el medio ambiente. Ecología agricola. Montevideo. 1937. 60 p. illus., maps, tables, diagrs. 29½ cm.

[Berlin. Zentrale für wissenschaftliches Berichtswesen über Luftfahrtforschung.]

Luftfahrt-Literaturschau 1938, Lieferung 7, Karte 8.361 bis 8.420. Berlin. 1938. [20] p. 32 cm.

Birkeland, Bernt Johannes.

Ältere meteorologische Beobachtungen in Ullensvang; Luftdruck und Temperatur seit 100 Jahren. [Oslo. 1932.] 40 p. incl. tables. 31 cm. [Geofysiske publikasjoner. v. ix. no. 6.]

Bjerknes, Jakob.

Exploration de quelques perturbations atmosphériques à l'aide de sondages rapprochés dans le temps. [Oslo. 1932.] 52 p. incl. illus. (charts), tables, diagrs. 2 fold. pl. (charts). 31 cm. [Geofysiske publikasjoner. v. ix. no. 9.]

Bjerknes, Vilhelm Friman Koren.

On the dynamics of the circular vortex with applications to the atmosphere and atmospheric vortex and wave motions. Kristiania. 1931. 88 p. diagrs. 31 cm. [Geofysiske publikationer v. ii. no. 4.]

Bliss, George S.

Believe it or not Philadelphia climate is generally fair. Philadelphia. 1938. p. 107–110. illus. 29½ cm. (From Frontiers, a magazine of natural history. v. 2, no. 4. April, 1938.)

Brooks, Charles F.

Hurricanes into New England. Meteorology of the storm of September 21, 1938. New York. [1939.] p. 119–127. maps, diagr. 28 cm. (Reprinted from the geographical review. v. 29, no. 1. Jan., 1939.)

Büttner, K.

Erythembildung durch Sonnen- und Himmelsstrahlung. [Berlin. 1938.] p. 610–615. diagr. 27 cm. (Sonderabdruck aus "Strahlentherapie." 61. Band. 1938.)

Carslaw, Horatio Scott.

Introduction to the theory of Fourier's series and integrals. 3d ed., rev. and enl. London. 1930. xiii, 368 p. diagrs. 22½ cm. "References" at end of each chapter.

Castens, Gerhard.

Klima, Witterung und Wetter in Deutsch-Ostafrika. Berlin. [1936?] p. 42–55. tables. 22½ cm. (Sonderabdruck aus Deutsch-Ostafrika gestern und heute, von oberstabsarzt Dr. Arning.)

Cattala, L.

Que sait-on des efforts du vent sur les bâtiments et les ouvrages d'art? Louvain. 1937. 44 p. illus., tables, diagrs. 25 cm. (Extrait de la Revue des questions scientifiques, mai 1937.)

Chosen. Weather bureau.

Results of the meteorological observations in Tyôsen for the lustrum, 1931–1935. Zinsen. 1936. 56 p. maps, tables. 31 cm.

Combier, Ch.

Vents de sable et pluies de boue. Mémoires de Ch. Combier, P. Gaubert, L. Petitjean. Notes de R. Beaulieu, R. Bochet, P. Habault, A. Lemieuvre, M. Mézin et G. Roche. Paris. 1937. 135 p. plates, maps, tables, diagrs. 28 cm. (Office national météorologique de France. Mémorial No. 27.)